

"JORDAN'S SCALAR STARS" AND DARK MATTER.

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Here we are starting the study of the field equations of relativistic scalar tensor theories in the spherically symmetric gravitational field. In the present article we shall consider as an example only the simplest Jordan-Brans-Dicke (JBD) one. To illustrate the property of the spherically symmetric JBD configuration we exhibit a new representation of the well-known four dimensional solutions. In this model, a suitable segment of Brans solution is chosen for the interior of the object while the outer region consists of a Schwarzschild vacuum. We have constructed "Jordan's scalar stars" model consisting of three parts: a homogeneous inner core with equation of state $p_M = \varepsilon \varrho_M$; an envelope of Brans spacetime matching the core and the exterior Schwarzschild spacetime. We have also showed that this toy model can explain the intergalactic effects without the dark matter hypothesis.

1. Introduction

The spirit of scalar-tensor extension of general relativity is an attempt to properly incorporate the Mach's principle [1] and Dirac's large number hypothesis [2] in which Newton's constant is allowed to vary with space and time. Apart from this, it is known that the Jordan-Brans-Dicke (JBD) scalar field plays the role of classical exotic matter required for the construction of traversable Lorentzian wormholes [3], [4]. The most prominent example of scalar-tensor theories of gravity is perhaps the JBD theory [5], [6]. These theories introduce a new fundamental scalar field which appears to be coupled non-minimally to gravity (in the so-called Jordan frame).

It is usually believed that when the effective JBD parameter ω is sufficiently large, the scalar-tensor theories of gravity are compatible with the solar system tests. However, a number of exact JBD solutions have been reported not to tend to the corresponding general relativity solutions [7], [8], [9]. These situation are alarming since the standard belief that JBD theory always reduces to general relativity in the large ω limit is the basis for setting lower limits on the ω -parameter using celestial mechanics experiments [10], [11], [12]. To make the situation worse, as showed by Hawking [13] and Johnson [14] (cf. Thorne and Dykla [15]) the only black holes in the JBD theory are Einstein black holes. If, following Hawking theorem [13], we make the reasonable demand that the solution of scalar-tensor theory field equations in empty space is the Schwarzschild solution lead to free estimates of lower limit of ω . Moreover, when the energy momentum tensor of ordinary matter vanishes, for all values of ω the JBD theory can

agrees with Einstein theory up to any desired accuracy and hence observations cannot rule out the JBD theory in favor of general relativity. Note, it is known that in empty space some vector-metric theories [16] can be recast simply in the Einstein's theory with $\vec{K} = 0$, but inside a matter the potential for the effective vector degree of freedom may play a complicated and nontrivial role [17]. It is thus imperative to study interior of relativistic stars in which case these theories could give different predictions. It is therefore important to study the situation more closely.

The paper is organized as follows. After giving a short account of the JBD theory, it has been shown in section 2 that one can find "Jordan's scalar stars" solutions in isotropic coordinates. The nature of the "Jordan's scalar stars" with perfect fluid matter core has been discussed in section 3. Finally the results are summarized in section 4.

2. The "Jordan's scalar stars" solutions.

There is the original spirit [5], [6] of JBD theory of gravity in which the scalar field ϕ is prescribed to remain strictly massless by forbidding its direct interaction with matter fields. However, the pure JBD theory can be thought of as a kind of theory having a non-canonical kinetic term and being coupled to gravity non-minimally. On the other hand, it is frequently argued that the spherically symmetric self-gravitating solitons appear in a number of field systems coupled to gravity. For example there are boson star solutions in the Einstein-Klein-Gordon system [30] and Einstein-Yang-Mills theory possesses the Bartnik-McKinnon solutions [31]. It is there-

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fore natural to ask whether a scalar star solutions might exist in the JBD theory. In fact one can regard the field equations as being simply the Einstein equations with a scalar field which interacts with all other matter fields through the trace of their energy momentum tensor.

The search for the exact spherically symmetric solutions is continuously of an interest to physicists. These models have been studied ever since the first solution of Einstein's field equation was obtained by Schwarzschild. Due to highly non-linear character of scalar-tensor gravitational theories, a desirable pre-requisite for studying strong field structure is to have knowledge of exact explicit solutions of the field equations [18]. The Birkhoff's theorem does not hold in the presence of JBD scalar field, hence several static solutions of the scalar-tensor theories seem possible in spherically symmetric vacuum case [19]. Notice that, it is obvious that we must apply the Birkhoff theorem even in Einstein theory only in the exterior vacuum domain outside the star.

Consider spherically symmetric spacetime geometry. The most common form of line element of a D-dimensional spherically symmetric spacetime in comoving coordinates can be written as

$$ds^2 = -g_{tt}(r, t) dt^2 + g_{rr}(r, t) dr^2 + \rho^2(r, t) d\Omega_{(D-2)}^2. \quad (1)$$

where $d\Omega_{(D-2)}^2$ is the line element on a unit D-2 sphere:

$$d\Omega_{(D-2)}^2 = [d\theta_{(0)}^2 + \sum_{n=1}^{D-3} d\theta_{(n)}^2 \left(\prod_{m=1}^n \sin^2 \theta_{(m-1)} \right)]. \quad (2)$$

One of the basic problems in the description of a source of gravitational field in relativistic theories is the choice of proper radial variable r . The physical and geometrical meaning of the radial coordinate r is not defined by the spherical symmetry of the problem and is unknown a priori [20], [21].

The forms of static spherically symmetric vacuum solution of the JBD theories are available in the literature often be explicitly written down in isotropic coordinates, defined by $\rho^2(r) = g_{rr}(r) r^2$. However, specific solutions, in general, do not

possess the symmetries of the equations they satisfy. The different gauge may describe different physical solutions of field equations with the same spherical symmetry [22]. As it was shown in [23], [24] a nonstandard gauge fixing for the applications of general theory of relativity to the stellar physics lead to solutions for some hypothetical objects with arbitrary large mass, density and size.

Scalar-tensor theories are described by the following action in the Jordan frame in D-dimensional space-time is:

$$S = \int d^D x \sqrt{-g} (\phi R - \omega(\phi) g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \lambda(\phi)) + S_m. \quad (3)$$

Here, R is the Ricci scalar curvature with respect to the space-time metric $g_{\mu\nu}$ and S_m denote action of matter fields. We use units in which gravitational constant $G=1$ and speed of light $c=1$. The dynamics of the scalar field ϕ depends on the functions $\omega(\phi)$ and $\lambda(\phi)$. It should be mentioned that the different choices of such functions give different scalar-tensor theories. We restrict our discussion to the JBD theory which characterized by the functions $\lambda(\phi) = 0$ and $\omega(\phi) = \omega/\phi$, where ω is a constant.

Variation of (3) with respect to $g_{\mu\nu}$ and ϕ gives, respectively, the D-dimensional field equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{\phi} T_{\mu\nu}^M + T_{\mu\nu}^{JBD}, \quad (4)$$

where

$$T_{\mu\nu}^{JBD} = \left[\frac{\omega}{\phi^2} \left(\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi \right) + \frac{1}{\phi} (\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \nabla_\alpha \nabla^\alpha \phi) \right]. \quad (5)$$

and

$$\nabla_\alpha \nabla^\alpha \phi = \frac{T_\lambda^{\lambda}}{(D-1) + (D-2)\omega}, \quad (6)$$

and T_λ^{λ} is the energy momentum tensor of ordinary matter which obeys the conservation equation $T_{\mu\nu;\lambda}^M g^{\nu\lambda} = 0$.

In this part of article we chose to work in static four dimensional isotropic spherically symmetric metric (1) with $\rho^2 = r^2 g_{rr}$,

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} (dr^2 + r^2 d\Omega^2). \quad (7)$$

because we found them to be the most widespread in literature and well known Brans solutions written down in this coordinates too. The use of isotropic coordinates is not a matter of deep principle and we do not rule out the possibility that there may still be other representations in other coordinate systems.

Hawking's theorem [13] in JBD states that the only spherically symmetric solution is static and given (up to coordinate freedom) by the Schwarzschild metric. However, as we have seen, even restricting to stationary spherically symmetry JBD theory has more solutions. Then one consequence of this is the possibility to use these solutions as interior and match it with Schwarzschild metric. To get a sense of the nature the static "Jordan's scalar star" solutions we consider here the simplest example, four-dimensional stars with Brans class I [6] solution as interior

$$\begin{aligned}\phi &= \phi_0 \left(\frac{1 - \frac{B}{r}}{1 + \frac{B}{r}} \right)^{\frac{C}{A}}, \\ \lambda &= \lambda_0 + \ln \left[\left(1 + \frac{B}{r} \right)^2 \left(\frac{1 - \frac{B}{r}}{1 + \frac{B}{r}} \right)^{\frac{A-C-1}{A}} \right], \\ \nu &= \nu_0 + \ln \left[\left(\frac{1 - \frac{B}{r}}{1 + \frac{B}{r}} \right)^{\frac{1}{A}} \right],\end{aligned}\quad (8)$$

where:

$$A = \sqrt{(C+1)^2 - C \left(1 - \frac{\omega C}{2} \right)},$$

and Schwarzschild solution as exterior of object

$$\begin{aligned}\phi &= 1, \\ \lambda &= \ln \left[\left(1 + \frac{\mu}{r} \right)^2 \right], \\ \nu &= \ln \left(\frac{1 - \frac{\mu}{r}}{1 + \frac{\mu}{r}} \right),\end{aligned}\quad (9)$$

In the case of ordinary stars model the discontinuity in the mass density at the surface entails via the field equations a jump in second derivations of metric coefficient, but first derivatives remains continuous so can be used to match to the vacuum solution. The spherical symmetry by itself automatically implies that once one calculates the Einstein tensor and goes to an orthonormal

frame. These comments are of course quite standard and in some form or another implicitly underlie all extant static spherically symmetric perfect fluid solutions. The same could happen for the "Jordan's scalar star" models also.

The JBD scalar field fluid, however, would fail to be a perfect fluid. That the stress-energy tensor of a scalar stars, unlike a classical fluid (but the similar as in the case of boson stars [32]), is in general anisotropic. For a spherically symmetric configuration, it becomes diagonal (5), i.e.

$$T_{\mu\nu}^{JBD}(\phi) = \text{diag}(\rho, -p_r, -p_\perp, -p_\perp).$$

In contrast to a neutron star, where the ideal fluid approximation demands the isotropy of the pressure, for spherically symmetric "Jordan's scalar star" there are different stresses p_r and p_\perp in radial or tangential directions, respectively.

Now in order to justify calling the geometry an exact solution we need an explicit definition for the constant in Brans solution. The integration constants of Brans solution C , λ_0 , ν_0 and B are arbitrary. However, it is possible to match this solution to the vacuum Schwarzschild metric. The brief computation yields.

$$\begin{aligned}B &= r_* \sqrt{\frac{2r_*^2 - 2r_*\mu + \mu^2(2+\omega)}{-2r_*\mu + 2\mu^2 + r_*^2(2+\omega)}}, \\ C &= \frac{2(r_*^2 - r_*\mu + \mu^2)}{r_*\mu\omega}, \\ \lambda_0 &= \ln \left(\frac{\left(1 - \frac{2B}{B+r_*} \right)^{\frac{2r_*^2 + 2\mu^2 + r_*\mu(\omega-2)}{A r_*\mu\omega}} (r_* + \mu)^2}{r_*^2 - B^2} \right), \\ \nu_0 &= \ln \left(\frac{\left(1 - \frac{2B}{B+r_*} \right)^{-\frac{1}{A}} (r_* - \mu)}{r_* + \mu} \right), \\ \phi_0 &= \left(\frac{r_* + B}{r_* - B} \right)^{\frac{2(r_*^2 + \mu^2 - r_*\mu)}{A r_*\mu\omega}}.\end{aligned}$$

where r_* the radius of "Jordan's scalar star" where we match the both solutions and the mass μ is defined as Keplerian mass, as seen by a distant observer.

Then in this context, unlike the scalar-tensor theory spirit of the original JBD gravity, the internal scalar field is not viewed as a part of the gravitational degrees of freedom but instead is thought of as playing the role of a matter degree of freedom. Obviously, this solution has a geometrical

topological nature and may be used in the attempts to reach description of "matter without matter". Apparently then, these models are also expected to provide a successful explanation for the phenomena associated with the dark matter.

Furthermore, there is now a growing consensus that wormholes are in the same chain of stars and black holes. A remarkable feature of our model is the fact that, one can represent it as wormhole the "bridges" between two separated Universes of different natures.

3. The "Jordan's scalar stars" with perfect fluid matter core.

On the other hand, the more difficult task is the construction of interior perfect fluid solutions which are of great astrophysical interest. In this line of thought, it is interesting to note that a relatively new model denoted as a gravastar (gravitational vacuum star) [33], consists of a compact object with an interior de Sitter condensate, governed by an equation of state given by $p = -\varrho$, matched to a shell of finite thickness with an equation of state $p = \varrho$. In this work, an extension of the gravastar picture is explored by matching an interior solution with $p_M = \varepsilon \varrho_M$ to an exterior Schwarzschild solution at a junction interface, comprising of a "scalar shell". Although this does not closely describe realistic stars, it can be adequate for indicating the behavior of mass limits and the stability properties of equilibrium configurations.

There are some known exact perfect fluid interior solutions in JBD [34], [35], [36]. Those solutions, however, are not physically acceptable: the pressure is singular at the center or the solutions have not a well defined boundary. Nevertheless, the exact solutions, even unrealistic, could qualitatively describe the case of a static, spherically symmetric perfect fluid "Jordan's scalar star".

In what follows we will consider the case of cold ultrahigh-density static configuration. One can use a perfect-fluid ordinary matter model with simple equation of state

$$p_M = \varepsilon \varrho_M.$$

Because of the considerations above we allow for three different regions with the three different equations of state,

Interior : $0 \leq r < r_1$, $p_M = \varepsilon \varrho_M$, $\phi \neq$ constant,

Shell : $r_1 < r < r_2$, $\varrho_M = p_M = 0$, $\phi \neq$ constant,

Exterior : $r_2 < r$, $\varrho_M = p_M = 0$, $\phi =$ constant.

At the interfaces $r = r_1$ and $r = r_2$, we require the scalar field ϕ and metric coefficients and first derivatives of metric coefficients to be continuous, although the derivatives of ϕ be able to discontinuous from the first order. Since our model is a mixed perfect fluid and exotic matter core, the requirement that pressure and density involve the scalar fields tell us that we can find the surface of the star by locating the first zero of total $p(r)$ or $\varrho(r)$.

The interior JBD solution for isotropic coordinates can be obtained by using the method discussed in [34]. The field equations (4), (6) can be integrated to give

$$\begin{aligned} \phi &= a e^{c \nu}, \\ \lambda &= \lambda_s - \frac{V \ln((1-b)r)}{b-1}, \\ \nu &= \nu_s + Q \ln r, \\ \varrho &= c e^{c \nu_s - 2\lambda_s} r^c Q^{-2} a Q ((1-b)r)^{\frac{2V}{b-1}} \times \\ &\quad \frac{((b-1)(1+Q+cQ)-V)(3+2\omega)}{(b-1)(3\varepsilon-1)}. \end{aligned} \quad (10)$$

where

$$\begin{aligned} c &= \frac{3\varepsilon-1}{(3+2\omega)+(\omega+1)(3\varepsilon-1)}, \\ b &= 1 - \frac{Q}{2} + \frac{c Q (2+\omega-\varepsilon(3+\omega))}{2(3\varepsilon-1)}, \\ V &= \frac{(Q-1)Q}{2} + \frac{c Q (2+\omega+Q\omega-\varepsilon(3+\omega))}{2(3\varepsilon-1)} + \\ &\quad + c Q^2 \frac{1-\varepsilon\omega+c\varepsilon(3+2\omega)}{2(3\varepsilon-1)}, \\ Q &= \frac{2\varepsilon(2+\omega+3\varepsilon(1+\omega))}{\sqrt{2+\omega+6\varepsilon(1+\omega)+9\varepsilon^2(2+\omega)}} \times \\ &\quad \frac{1}{\sqrt{2+\omega+6\varepsilon(1+\omega)+\varepsilon^2(6+\omega)}} \end{aligned}$$

Consequently possibility of composite models is obtained by matching the surfaces $r = r_1$ and $r = r_2$ since solutions would be forced to match on both surfaces. We examine matching across these surfaces between solutions (10), (9) and (9) to obtain the values of constants of integration. After some algebra one can decide

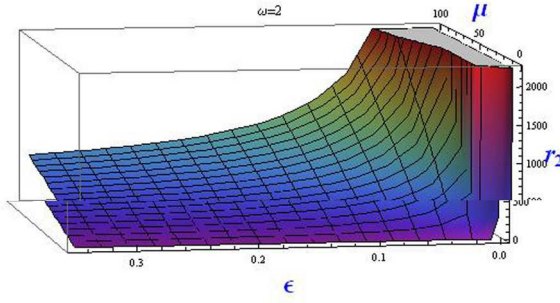


Figure 1: The outer radius of the scalar envelope is r_2 and the total Keplerian mass inside r_2 is μ . The equation of state of the core is $p_M = \varepsilon \varrho_M$.

$$B = \frac{(1 - C + A^2) \mu}{2A} + \frac{\sqrt{C^4 + (A^2 - 1)^2 - 2C^2(1 + A^2)} \mu}{2A},$$

$$C = -\frac{2A - (b - 1)(2 + Q)}{(b - 1)Q\omega} - \sqrt{W}$$

where

$$W = \frac{4Q\omega + (2 + Q)^2}{Q^2\omega^2} + \frac{2A^2(2 + \omega)}{(\beta - 1)^2 Q^2\omega^2} - \frac{4A(2 + Q + \omega - Q\omega)}{(b - 1)Q^2\omega^2}$$

In a realistic situation, interior matter is present inside a star. Then, to obtain a viable stellar configuration, one has to match solutions on both surfaces of scalar shell. Since there are the freedom of determination integration constant between inside and outside the star, and hence it is easier to satisfy the matching condition. Indeed, the matching mechanism has been shown by us to work in this model. It is clear from this fact that one can construct viable models of "Jordan's scalar star" with ordinary perfect fluid equation of state.

The above analysis shows that, for certain values of $\varepsilon < 0.3$, the solutions (10) may actually be interpreted to yield a gravitation interaction inwardly of objects goes to zero in the centre and increases beside surfaces. This choice of the coefficient ε represents a monotonic increasing "gravitation constant" in the star interior, and was obtained previously in the analysis of static spheres [37] as a specific case of the Newtonian limit of JBD theory.

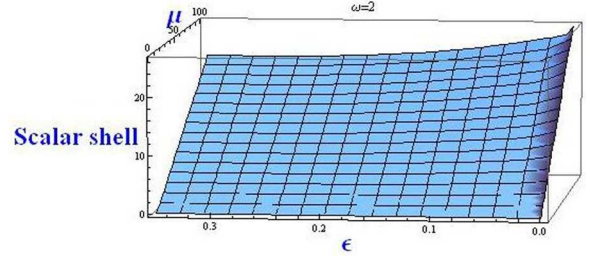


Figure 2: Sizes for shells and Keplerian masses in the "Jordan's scalar stars".

4. Discussion and Conclusions

Using the key assumption that the Brans class I [6] solution physically acceptable because the solutions have a well defined boundary and they can match with the Schwarzschild exterior solution at the boundary surface it was found the mass and radius of scalar stars. On the other hand this solution can be interested in giving a pure field representation of particles.

The new results include matching between exact interior solutions in the perfect fluid family and the Brans and after that Schwarzschild solutions. It is clear that the approach to scalar stellar structure, developed in the present article, calls for revision of some of widely accepted features of the relativistic theories of stars. The changes are not based on the critics of these theories, but on more deep understanding of its applications, and on attempt to solve some open problems. Hence, our toy models have an essential impact only on the theory of the interior of relativistic stars, and on theory of spreading of different physical fields in stars, and around the stars. Among others, this solution one can assume as wormhole the "bridges" between separated Schwarzschild and Brans Universes.

In this context, it is demonstrated that our toy model can successfully predict the emergence of dark matter in terms of a self-gravitating space-time solution to the JBD field equations with non-trivial energy density of the JBD scalar field which was absent in the context of general relativity where the Newton's constant is strictly a constant having no dynamics. Bearing the above evaluation in mind, let us comment on the other specific models of scalar and vector metric gravity. The some vector-metric theories one shared the similar structure of energy momentum tensor [16]. Therefore, we expect that the same problem arises in these theories. We have shown that this predicts a rather interesting physics for the range

from stars to clusters of galaxies. First of all we point out that in this model the stars acquires features of a two-component objects (ordinary matter and scalar or vector field) whose distribution in the observed 3-dimensional volume can has, in an addition to standard model, an envelope of scalar or vector fields. Moreover, such a picture can represents a Schwarzschild background, while the interior should be considered as vacuum solution of scalar or vector-metric theories which defined a Keplerian mass of this object.

Some final remarks, now the local value of the Newtonian "gravitation constant" measured only near the Earth. For central and peripheral parts of Galaxy value of "gravitation constant" can be vastly differ from Newtonian value. In this point of view the dark matter problem may be explain by a mixture of various interacting scalar and vector field potentials inside the galaxies and galaxies clusters.

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6. Appendix

The possibility that spacetime has more than four dimensions, was first contemplated by Nordström [25]. It is helpful here to emphasize the equivalence between the $(D + 1)$ Kaluza-Klein theories with empty D -dimensional JBD theories when $\omega = 0$ [26]. Now the idea that spacetime has extra dimensions lies today at the heart of the most theories of unification of the fundamental interactions present in Nature, but the introduction of the fifth and higher dimensions requires a careful approach. In particular, static spherically symmetric vacuum solutions in D -dimensional scalar-tensor theories shed new light on the complex features of objects in these models.

According to the standard textbooks [27] the static spherically symmetric vacuum JBD solutions can often not be explicitly written down in standard Hilbert or curvature coordinates and it is better to work in isotropic coordinates. However, the first exact solution of JBD field equations in widespread Hilbert coordinates were obtained in parametric form by Heckmann [28], soon after Jordan proposed scalar-tensor theory. This solution describes the geometry of the space-time

exterior to a perfect fluid sphere in hydrostatic equilibrium. The change of variables technique for obtaining the similar static solutions in D dimensions are known by now [29]. Choose the static spherically symmetric metric (1) in Hilbert gauge: $\rho = \hat{r}$, $g_{rr} = e^{2\lambda(\hat{r})}$ and $g_{tt} = e^{2\nu(\hat{r})}$,

$$ds^2 = -e^{2\nu(\hat{r})} dt^2 + e^{2\lambda(\hat{r})} d\hat{r}^2 + \hat{r}^2 d\Omega^2. \quad (11)$$

It is actually possible to change variables so that one will replace variable \hat{r} by $\hat{r}(\nu)$ then the field equations (4), (6) take a form:

$$-1 + \lambda' - \frac{(D-2)\hat{r}'}{\hat{r}} + \frac{\hat{r}''}{\hat{r}'} = \frac{\phi''}{\phi'}, \quad (12)$$

$$-1 + \lambda' - \frac{(D-2)\hat{r}'\lambda'}{\hat{r}} + \frac{\hat{r}''}{\hat{r}'} = \quad (13)$$

$$-\frac{\phi'\lambda'}{\phi} + \frac{\omega\phi'^2}{\phi^2} - \frac{\phi'\hat{r}''}{\phi\hat{r}'} + \frac{\phi''}{\phi},$$

$$-1 + \lambda' - \frac{(D-3)(-1+e^{2\lambda})\hat{r}'}{\hat{r}} = \frac{\phi'}{\phi}, \quad (14)$$

$$-1 + \lambda' - \frac{(D-2)\hat{r}'}{\hat{r}} + \frac{\hat{r}''}{\hat{r}'} = \frac{\phi'}{\phi}. \quad (15)$$

where now ν is a new variable and the primes denote derivatives with respect to ν .

By eliminating $\hat{r}(\nu)$ and $\lambda(\nu)$ from equations (12) and (15) we can obtain the following equation

$$\frac{\phi''}{\phi'} - \frac{\phi'}{\phi} = 0. \quad (16)$$

Eq. (16) can immediately be integrated to give

$$\phi = \phi_0 e^{\nu b}. \quad (17)$$

where ϕ_0 and b are an arbitrary constants of integration. Specifically, if the scalar field is constant ($\phi = \text{const}$) then the solution of Eqs. (12) - (15) is a D -dimensional Schwarzschild solution.

$$\lambda = -\nu, \quad (18)$$

$$\hat{r} = (-1 + e^{2\nu})^{\frac{1}{3-D}}.$$

After a straightforward calculation using Eqs. (12) - (15) and (17), we obtain the three possible solutions for metric components and function $\hat{r}(\nu)$:

$$\lambda = \ln \left(\sqrt{\frac{A}{A + (1+b)^2}} \sec \left[\sqrt{A} (q + \nu) \right] \right), \quad (19)$$

$$\hat{r} = a \{ (D-3) e^{(1+b)\nu} ((1+b) \times \cos \left[\sqrt{A} (q + \nu) \right] + \sin \left[\sqrt{A} (\gamma + \nu) \right]) \}^{\frac{1}{D-3}}$$

$$\lambda = \ln \left(\sqrt{\frac{A}{A + (1+b)^2}} \sec \left[\sqrt{A} (q + \nu) \right] \right), \quad (20)$$

$$\hat{r} = a e^{-\frac{(1+b)(\gamma+\nu) + \operatorname{arccoth} \left[\frac{(1+b) \cot \left[\sqrt{A} (q + \nu) \right]}{\sqrt{A}} \right]}{D-3}} \times \{ (D-2) (A - (1+b)^2) - (A + (1+b)^2) \cos \left[2\sqrt{A} (q + \nu) \right] \}^{\frac{1}{2D-5}}$$

$$\lambda = \ln \left(2\sqrt{-A} a e^{\nu\sqrt{-A}} \right) - \ln((1+b) - \sqrt{-A} - e^{2\nu\sqrt{-A}} ((1+b) + \sqrt{-A}) a), \quad (21)$$

$$\hat{r} = a \left(\frac{\sqrt{-A}}{(D-3) e^{((1+b)-\sqrt{-A})\nu}} \right)^{\frac{1}{D-3}} \times \left(\frac{2}{-1 + q e^{2\nu\sqrt{-A}}} \right)^{\frac{1}{D-3}},$$

where q and a are an arbitrary constants of integration and

$$A = \frac{(b-1)^2 (1+2\omega) - (D-1) (1+b^2 (1+\omega))}{D-2}$$

The derivation extends previous results of Heckmann for higher dimensions. By choosing some values of arbitrary constant b several special classes of static vacuum solutions can be obtained in the framework of the scalar-tensor model. Assuming that the arbitrary constant $b = -1$, one can obtain another solution:

$$\phi = \phi_0 e^{-\nu}, \quad (22)$$

$$\lambda = \frac{1}{2} \ln \left[-\sinh \left(\frac{\nu \sqrt{(\omega+2)(D-3)}}{\sqrt{D-2}} - q \right)^{-2} \right],$$

$$\hat{r} = a \cosh \left[q - \frac{\nu \sqrt{(\omega+2)(D-3)}}{\sqrt{D-2}} \right]^{\frac{1}{3-D}}$$

Alternatively, instead of taking the $b = -1$, we can form the constant $b = 2/\omega$ and obtain an different description of the solution.

$$\begin{aligned} \phi &= \phi_0 e^{\frac{2\nu}{\omega}}, \\ \lambda &= q - \frac{\nu(2+\omega)}{\omega} \\ \hat{r} &= a \left(-e^{2q} + e^{\frac{2\nu(2+\omega)}{\omega}} \right)^{-\frac{1}{D-3}} \end{aligned} \quad (23)$$

Now using the constant from the equation (17), viz.

$$b = \pm \frac{1 - \sqrt{(3-D)(\omega(D-2) + D-1)}}{D-2 + \omega(D-3)}$$

we get

$$\begin{aligned} \phi &= \phi_0 e^{\nu b}, \\ \lambda &= -\ln[B\nu + q] \\ \hat{r} &= a (e^{B\nu} (-1 + q + B\nu))^{\frac{1}{3-D}} \end{aligned} \quad (24)$$

So far the solutions found is a simple mathematically consistent solutions. It was constructed to clarify the method described in this section. In order to obtain a physically acceptable solution, it is necessary to carry out a more careful analysis.

We have explicitly characterized the D - dimensional spacetime metrics corresponding to the JBD static spherically symmetric geometries in a relatively straightforward manner. Although a tremendous amount is already known concerning static spherically symmetric spacetimes the particular approach adopted in the present article may be useful for understanding the inherent non-linear character of JBD gravitational theory.

While the vacuum JBD field equations possess a well-known D - dimensional Schwarzschild solution for an isolated mass M , one can transform other above vacuum spherically symmetric solutions into an interior of "Jordan's scalar star". If we consider models in which the central region contains the scalar field only, then it can be used to study the interior structure of the D - dimensional relativistic objects with anisotropic pressure of exotic matter.

References

- [1] E. Mach, The Science of Mechanics (Open Court, LaSalle, IL, 1960). Mach's Principle: From Newton's Bucket to Quantum Gravity, edited by J. Barbour and H. Pfister (Birkhauser, Boston, MA, 1995).

- [2] Dirac, P. A. M., Long range forces and broken symmetries, Proc. R. Soc. Lond. A333, 403 (1973)
- [3] M. Visser, Lorentzian wormholes: from Einstein to Hawking, Springer-Verlag, New York, Inc. (1996).
- [4] K. K. Nandi, B. Bhattacharjee, S. M. K. Alam, J. Evans, Brans-Dicke wormholes in the Jordan and Einstein frames, Phys. Rev. D 57, 823 - 828 (1998); A. Bhadra, K. Sarkar, Wormholes in vacuum Brans-Dicke theory, arXiv:gr-qc/0503004, (2005).
- [5] P. Jordan, *Schwerkraft und Weltall*, Vieweg (Braunschweig) 1955.
- [6] C. Brans and R. H. Dicke, Mach's Principle and a Relativistic Theory of Gravitation, Phys. Rev. 124, 925-935, (1961).
- [7] T. Matsuda, On the Gravitational Collapse in Brans-Dicke Theory of Gravity, Progr. Theor. Phys. 47, 738-740, (1972).
- [8] C. Romero and A. Barros, Brans-Dicke vacuum solutions and the cosmological constant: A qualitative analysis, Gen. Rel. Grav. 25, 491-502 (1993).
- [9] M.A. Scheel, S.L. Shapiro and S.A. Teukolsky, Collapse to black holes in Brans-Dicke theory. II. Comparison with general relativity, Phys. Rev. D 51, 4236 - 4249 (1995).
- [10] C. M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 9 (2006), [http:// www.livingreviews.org/lrr-2006-3](http://www.livingreviews.org/lrr-2006-3)
- [11] Y. Fujii, K. Maeda, *The ScalarTensor Theory of Gravitation*, Cambridge University Press, (2003).
- [12] T. M. Eubanks et al. (1999). Advances in solar system tests of gravity. In: Proc. of The Joint APS/AAPT 1997 Meeting, 18-21 April 1997, Washington D.C. BAAS, Online preprint, <ftp://casa.usno.navy.mil/navnet/postscript/prd.15.ps>.
- [13] S. W. Hawking, Black holes in the Brans-Dicke Theory of gravitation, Commun.Math. Phys. 25, 167171 (1972).
- [14] M. Johnson, Lett. Nuovo Cimento 4, 323327 (1972)
- [15] K.S. Thorne, J.J.Dykla, Black Holes in the Dicke-Brans-Jordan Theory of Gravity, Ap. J. 166, L35L38 (1971)
- [16] C. M. Will, *Theory of experiment in Gravitational Physics* (Camb. Univ. Press, Cambridge, 1993).
- [17] V. Bashkov, S. Kozyrev, Problems of high energy physics and field theory, 22, Protvino, (1991).
- [18] A. Bhadra, K. Sarkar, On static spherically symmetric solutions of the vacuum Brans-Dicke theory arXiv:gr-qc/0505141 (2005).
- [19] J. O'Hanlon, B. O. J. Tupper, Vacuum-field solutions in the brans-dicke theory, Il Nuovo Cimento B (1971-1996), V. 7, N. 2, 305-312, (1972).
- [20] J. L. Synge, *Relativity: The General Theory*, North Holland Publ. Comp., Amsterdam, 1960.
- [21] A. S. Eddington, *The mathematical theory of relativity*, 2nd ed. Cambridge, University Press, 1930 (repr.1963).
- [22] P. P. Fiziev, Gravitational Field of Massive Point Particle in General Relativity, arXiv :gr-qc/0306088 (2003).
- [23] P.P.Fiziev, Novel Geometrical Models of Relativistic Stars. arXiv :astro-ph/0409456 (2004)
- [24] J. M. Aguirregabiria, Ll. Bel, Extreme objects with arbitrary large mass, or density, and arbitrary size arXiv :gr-qc/0105043, Gen. Rel. and Grav. 33, 2049 (2001).
- [25] G. Nordström, "Über die Möglichkeit das elektromagnetische Feld und das Gravitationsfeld zu vereinigen," Phys. Zeit. 15, 504506 (1914).
- [26] S. Rippl, C. Romero, R. Tavakoli, D-Dimensional Gravity from (D + 1) Dimensions, gr-qc/9511016,(1995)
- [27] I. K. Wehus, F. Ravndal (2006). Gravity coupled to a scalar field in extra dimensions. Preprint arXiv:gr-qc/0610048v2
- [28] O. Heckmann, P. Jordan, R.Fricke, *Astroph., Zur erweiterten Gravitationstheorie*, Z. 28, 113-149, (1951).
- [29] S. Kozyrev, A D-dimensional Heckmann-like solution of Jordan-Brans-Dicke theory, arXiv: 0712.2894v1 [gr-qc],(2007)

- [30] R. Ruffini and S. Bonazzola, "Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State". Phys. Rev. 187: 1767-1783, (1969).
- [31] S. Volkov, D. Gal'tsov, Gravitating Non-Abelian Solitons and Black Holes with Yang-Mills Fields, Physics Reports, 319, 1, 1-83, (1999); hep-th/9810070 (1998).
- [32] F. E. Schunck E. W. Mielke, Topical review, General relativistic boson stars, Class. Quantum Grav. 20 R301-R356 (2003), arXiv:0801.0307 [astro-ph], (2008).
- [33] P. O. Mazur and E. Mottola, "Gravitational Condensate Stars: An Alternative to Black Holes," [arXiv:gr-qc/0109035];
- [34] W. Bruckman, E. Kazes, Properties of the solutions of cold ultradense configurations in the Brans-Dicke theory, Phys. Rev. D 16, 261-268 (1977)
- [35] S. Kozyrev, Properties of the static, spherically symmetric solutions in the Jordan-Brans-Dicke theory, gr-qc/0207039
- [36] S. S. Yazadjiev, Interior perfect fluid scalar-tensor solution, gr-qc/0312019 (2003)
- [37] V.I.Bashkov, S.M.Kozyrev, Dark matter an effect of gravitation permeability of material in Jordan, Brance - Dicke theory, arXiv :gr-qc/0103009 (2001).